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# CONCRETE COMPOSITE CROSS-SECTION RESPONSE FOR PUSHOVER ANALYSIS

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## **Abstract**

Nonlinear pushover analysis has gained significance popularity and acceptance for evaluating the performance of structures, especially buildings, both for seismic and non-seismic loads. The pushover analysis relies on the non-linear cross-section response parameter as the basic input for determining the load-deformation and performance curves. Many software that carryout the pushover analysis, have the capability to generate such parameter for simple section and material models. However for complex cross-sections, material models and behavior, computing these parameters often needs to be done separately and then provided as an input to the analysis programs.

This paper presents a unified approach to determine the cross-sectional parameters needed for carrying out pushover analysis. A general, composite cross-section model is considered that includes the effects of pre-stress, differential creep, shrinkage, confinement, staged construction, strain hardening, tension stiffening, relaxation and bond slippage etc. Methodology is proposed to handle all of these effects is a single consistent computational procedure. The response that can be obtained from these computations is the capacity at specified failure criteria, the moment-curvature curves, moment rotation curves, stiffness properties, ductility ratio etc. The proposed model can be used for beams, columns and shear walls for new construction as well as for strengthened and retrofitted members.

## **1. INTRODUCTION**

Pushover analysis is a static, nonlinear procedure in which the magnitude of the structural loading is incrementally increased in accordance with a certain predefined pattern [1]. With the increase in the magnitude of the loading, weak links and failure modes of the structure are found. The procedure also helps to identify ductility requirements for various members. The loading is monotonic with the effects of the cyclic behavior and load reversals being estimated by using a modified monotonic force-deformation criteria and with damping approximations. Static pushover analysis is an attempt by the structural engineering profession to evaluate the real strength of the structure and it promises to be a useful and effective tool for performance based design. Pushover analysis is most suitable for determining the performance, especially for lateral loads such as earthquake or even wind. Since the structures do not respond as linearly elastic systems during strong ground shaking,

the linear analysis can not give true picture of structural system behavior. Non-linear Pushover analysis gives improve understanding of building behavior and provides more accurate prediction of global displacement, more realistic prediction of earthquake demand on individual components and elements and more reliable identification of “bad actors”. Pushover analysis is most advanced state of the practice because in Pushover analysis design is based on Expected Performance rather than on Ultimate Strength. The basic Ultimate Strength does not tell us what will be performance of the structure at Ultimate Capacity.

**1.1 Performance Based Design**

Performance is generally of concern for lateral loads such as earthquake and wind. The main factor that affects performance is the ductility of the members on the critical load path. In frame structures, the design of the joints between columns and beams is critical. The performance of shear walls is also of great importance for lateral load demands. The overall relationship of various response curves relevant to the performance based design or evaluation of structures using the Pushover analysis is shown in Figure 1. This figure shows how the material stress-strain curves contribute to the Moment-Curvature curve, which in turn affects the moment rotation behavior of plastic hinges, that in turn affect the structure performance curve and finally relates to the seismic demand curve defined in Accelerate Displacement Response Spectra (ADRS) format [1]. It can be seen that the Moment-Curvature curve is the basic cross-section response that must be determined to carry on this type of analysis. The Moment-Curvature curve and the corresponding moment rotation curve are generated after basic cross-section design is completed.

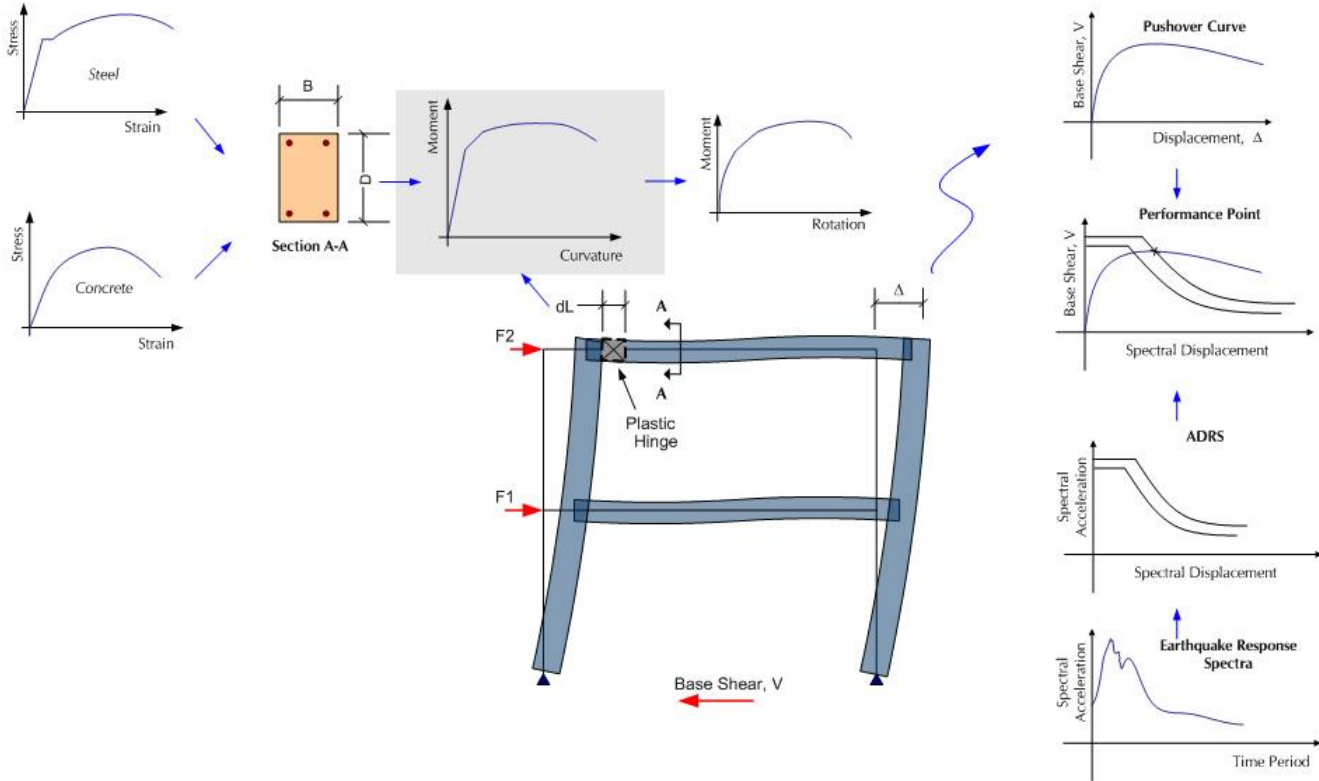


Figure 1: Moment-curvature curve as one of the key inputs in performance based design using Pushover analysis

## 2. ACTION DEFORMATION CURVE

It is well known that a structural member when loaded or subjected to actions undergoes deformations. The load deformation curve exhibits the entire behavior of the member right from initial loads to final loads or initial deformations to final deformations. A typical load deformation curve (Figure 2) can be divided into at least four distinct points can be identified on this curve. Broadly speaking, these points are:

- i) The point up to which the relationship between load and deformation can be considered nearly linear and the deformations are relatively small. This point is marked as 'A' on the curve.
- ii) The point at which the deformation starts to increase suddenly, at more or less constant load value or with relatively small increase in the load. This point is marked as 'B' on the curve.
- iii) The point at which the load value starts to drop with increasing deformations (point 'C').
- iv) And finally the point where load value become nearly zero and member loses all capacity to carry any loads and collapses or fails completely (point 'D').

The actual shape of the load deflection curve and relative locations of these points depends on many factors including cross-section geometry, material properties, slenderness considerations, type of deformation and corresponding load being considered.

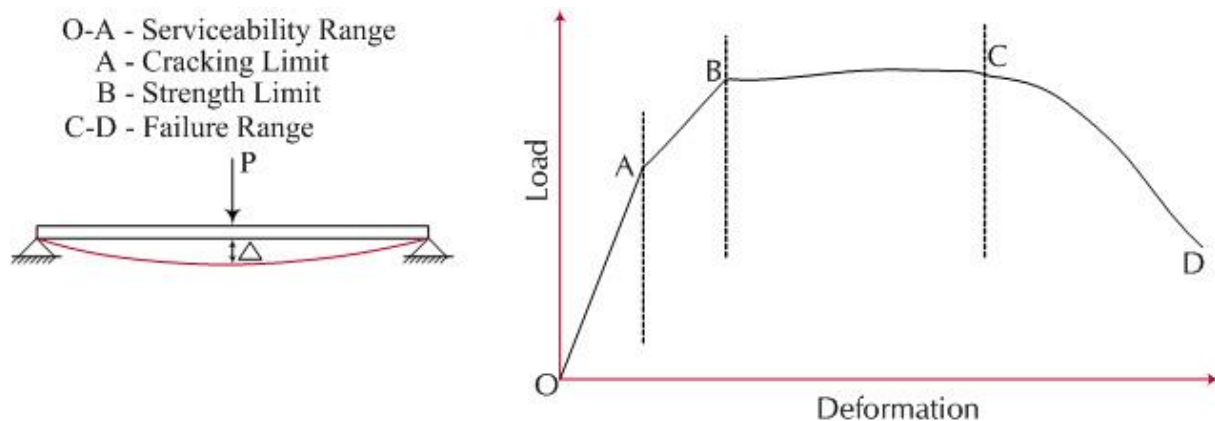
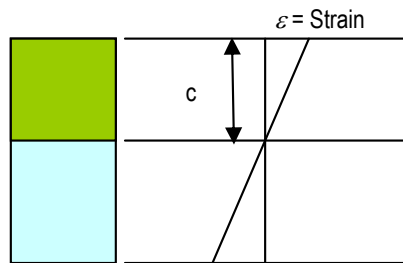


Figure 2: Various design states can be determined or related to points on the load-deformation curve

The load-deformation curves can be plotted between axial load and axial shortening, shear force and shear deformation, moment and curvature, and torsion and twist. Of all these curves, the moment-curvature curve is probably the most important curve for flexural design of beams, columns, shear walls and consequently for building structures. It is also probably the least understood or utilized in normal design practice. Many of the design codes and design procedures or design handbooks do not provide sufficient information for computation and use of these curves. Basically, curvature can be defined in several contexts. In geometry, it is rate of change of rotation. In structural behavior, curvature is related to moment through stiffness. For a cross-section undergoing flexural deformation, it can be computed as the ratio

of the strain to the depth of neutral axis. The curvature is simply the inverse of the radius of curvature that can be obtained from the slope of the strain profile (Figure 3). Significant amount of information can be obtained from Moment-Curvature curve to compute response quantities such as [2]:



Curvature =  $\epsilon / C$  (radian / unit length)

Figure 3: The definition of curvature in a cross-section

- Yield point, signifying the design strength
- Failure point, signifying the maximum deformation and load capacity
- Ductility ratio as a measure of performance
- Effective stiffness at a particular moment or rotation value
- Crack width for a given crack spacing
- Rotation of a finite length of hinge for a given moment
- Deflection of a finite length of member for a given moment
- Strain at any level across the cross section for a given moment

This information is very useful for non-linear analysis of structure including the post-elastic behavior. The Moment-Curvature relationship is also the basis for the capacity based, or Performance Based Design methods, especially useful in the analysis of structures using the Static Pushover Method, and in determining the rotational capacity of plastic hinges formed during high seismic activity. It is interesting to note that the Moment-Curvature curves obtained from the stress resultants are independent of the member geometry or the moment diagram, and for a given axial load is a property of the cross-section, just like the capacity interaction surface.

### 3. DETERMINING CROSS SECTION RESPONSE

In general cross-sections can be characterized and categorized in many ways [3]. Some of many classifications as below:

#### Based on Material

- Un-reinforced, Reinforced, Prestressed Concrete
- Fiber Reinforced Concrete, Ferrocement
- Steel-Concrete Composite
- Hot Rolled Steel, Cold formed Steel
- Structural Timber

#### Based on Geometry

- Circular, Elliptical, Rectangular, Flanged, Polygonal
- Hollow, Solid, Voided
- Symmetrical, Unsymmetrical, Asymmetrical

#### Based on Compressed Zone

One type of characterization is based on the dimensions and stiffness of the part of the section that is in compression. This characterization is based on the relative proportions of various geometric components of the section, especially when subjected to compressive strain.

This classification is generally applicable to steel section and includes:

- Slender Sections/Thin Walled Sections
- Compact Sections
- Plastic Sections

The cross-section of a member is the basic design component. In fact the one of the main purpose of the design process is to proportion the cross-sections effectively and economically. Any attempt to integrate the design approaches must begin at the cross-section level.

Let us consider a general cross-section that can be used to represent an un-reinforced, reinforced, prestressed and composite concrete beam-column. This cross-section is defined in terms of a set of closed areas and some points as shown in Figure 4.

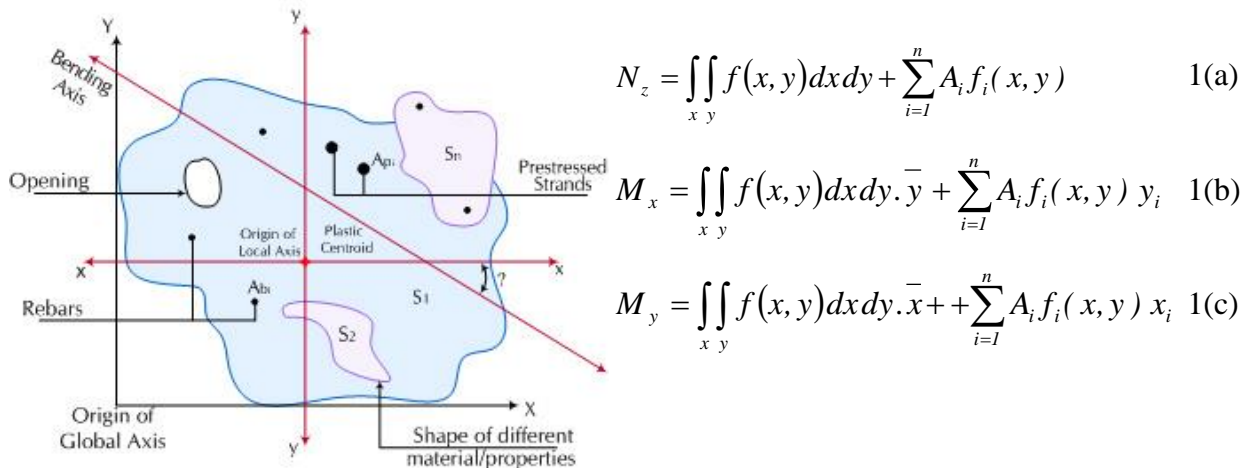


Figure 4: A generalized representation of cross-section having several shapes and materials

This cross-section is assumed to undergo a combined axial shortening and rotation about an axis often termed, as the neutral axis. A set of simple equilibrium equations 1(a-c) can be written for such a cross-section based on integration of stresses in various parts of the cross-section. The stresses in rebar types of components can be simply added [3, 4]. These equations can be reduced to two equations if the cross-section is oriented in such a way that the bending axis corresponds with Y-axis as shown in Figure 5.

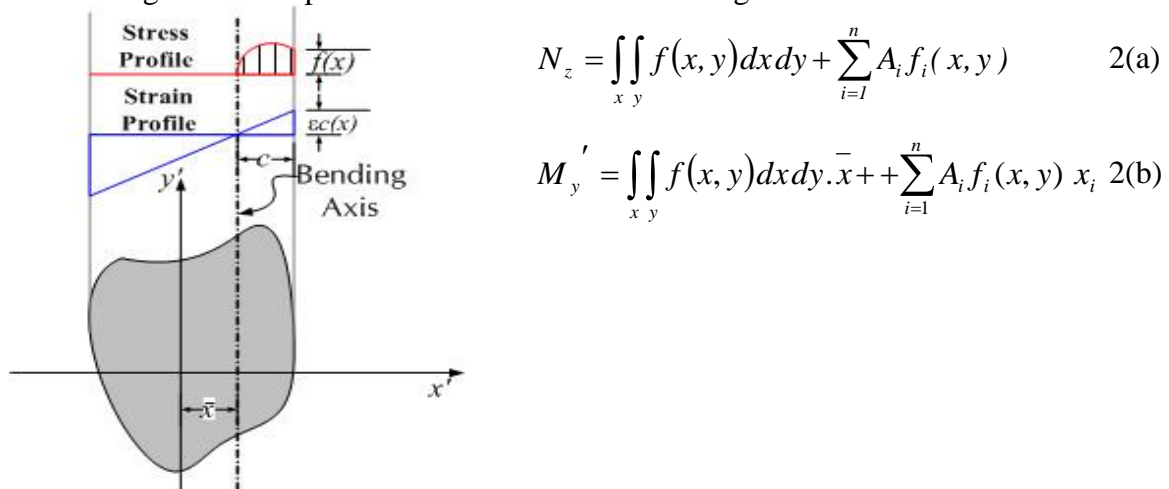


Figure 5: A transformed representation of cross-section, with bending axis parallel to y-axis

This moment  $M_y'$  in equation 2(b) is in fact the principal moment or the resultant moment. These two equations 2a and 2b give us the basis to compute the axial load and corresponding bending moment for a particular orientation, location and rotation of neutral axis or bending axis. The rotation of neutral axis depends on the depth ( $c$ ) and the strain on the compression fiber  $\epsilon$ . With the appropriate computing algorithm, these equations can be used to generate the entire response of the cross-section, especially the capacity surface shown in Figure 6(a) and the Moment-Curvature curve Figure 6(b) [5, 6].

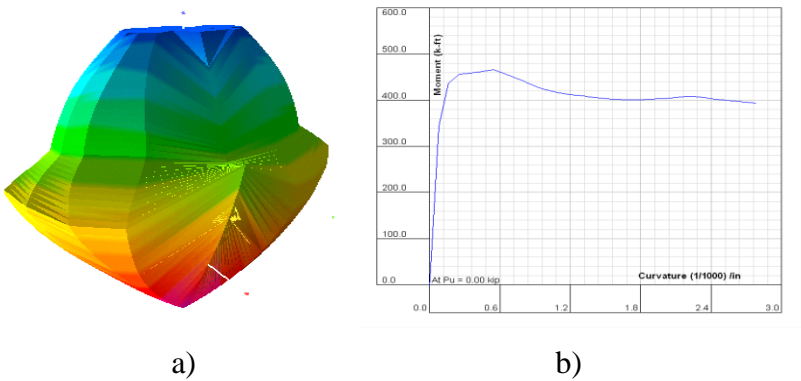


Figure 6: A typical a) capacity interaction surface and b) Moment-Curvature curve

The use of these equations provides us with the means to carry out the design and investigation of cross-sections as shown in Figure 7.

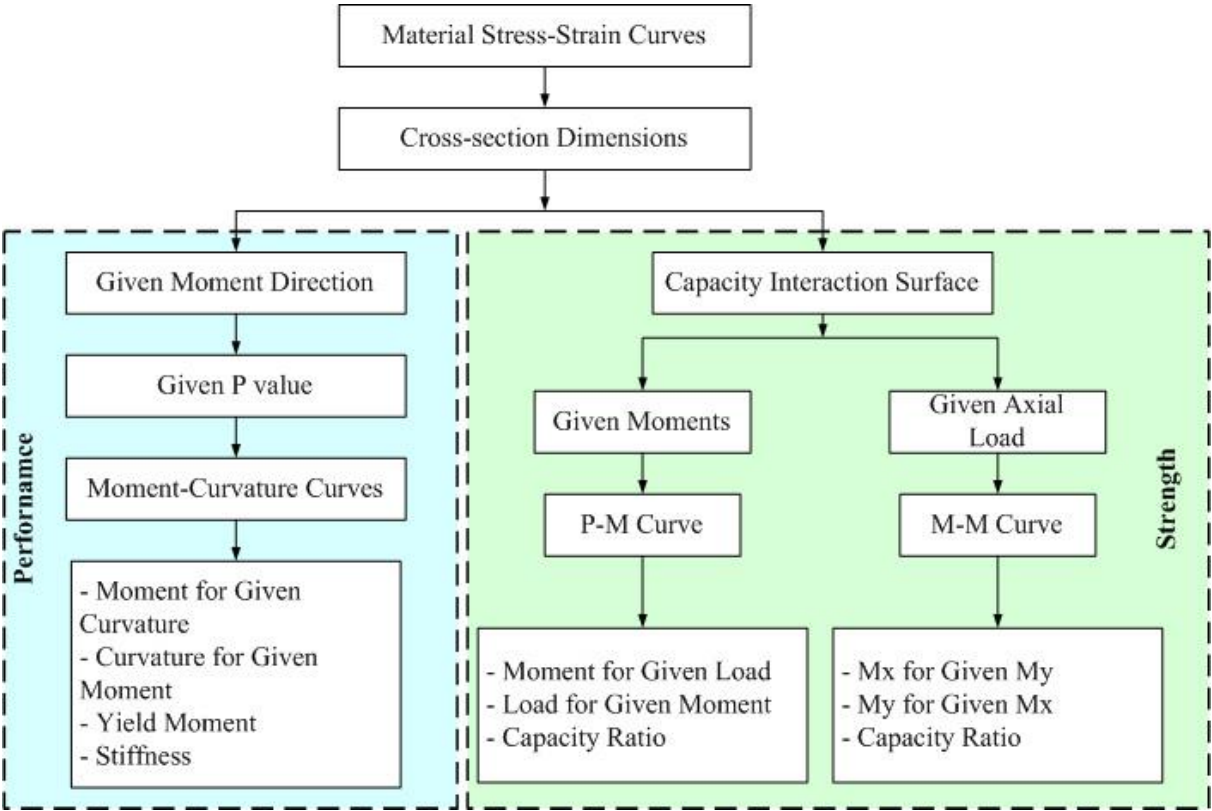


Figure 7: Determining cross-section response



### 3.1 Generation of Moment-Curvature Curves

The main idea behind generation of the Moment-Curvature curve is to obtain the neutral axis depth and the corresponding strain at the compression extreme for a given set of axial load and moment. There is no direct solution possible and an iterative approach needs to be used. It is often easier to fix the strain first and iterate on the depth of neutral axis, compute moment and axial force using Equation 2, until equilibrium with the axial load is achieved. The corresponding moment capacity at that depth of neutral axis and strain level is then used, along with the curvature at that point. As defined earlier, the curvature is simply the ratio between the strain and the depth of neutral axis. This curvature is measured in the units of radians/length units used to define the neutral axis depth. Once one moment-curvature set is obtained, the extreme fiber strain is changed and another solution is attempted to obtain yet another pair of moment and curvature. Several points are computed, using a small strain increment to plot a smooth curve. The generation of Moment-Curvature curve can be terminated based on any number of specific conditions such as, the maximum specified strain is reached, the first rebar reaches yield stress or any other strain level, the concrete reaches a certain strain level.

Also, during the generation of the Moment-Curvature curve the failure or key response points can be recorded and displayed on the curve. The axial load-shortening curve can be generated in a manner similar to that described for the generation of Moment-Curvature curve. However, in this case the iteration to determine the depth of neutral axis is not needed, as the neutral axis is assumed to be horizontal, in the absence at any moment. The strain is incremented and at each increment of strain, the corresponding axial load is determined using the appropriate material models

### 4. DUCTILITY AND STIFFNESS

One of the most important responses that affect the performance is the ductility, both at cross-section and member level. Ductility can be defined as the “ratio of deformation at a given stage to the maximum deformation capacity” [7]. Normally ductility is measured from the deformation at design strength to the maximum deformation at failure as shown in Figure 8. The most important factor effecting ductility of reinforced concrete cross-section is the

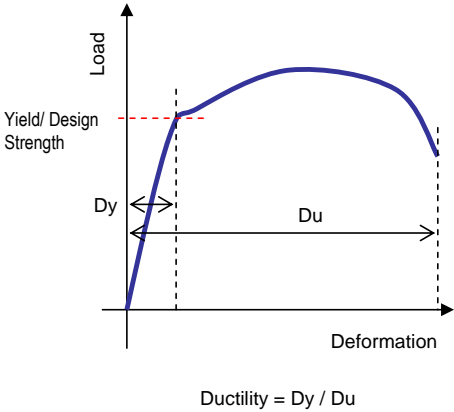


Figure 8: The definition of ductility using action deformation curve

confinement of concrete, the amount of confinement steel and the shape of confinement steel. Other factors include presence of axial load, stress-strain curve of rebars, amount of rebars in tension, amount of rebars in compression, and the shape of cross-section [7]. While ductility is important in the post yield response, stiffness is of importance throughout the response history. The stiffness property affects the performance of the structure. The concept of stiffness appears in almost all the relationships involving the structural response. The stiffness can be defined in many ways; however, the definition that we will use here means the *resistance to*



**deformation.** This resistance to deformation is exhibited by the structures at many levels as shown in Figure 9.

The overall resistance of the structures to over all loads is called the *Global Structure Stiffness*. This is derived from the sum of stiffness of its members, their connectivity and the boundary or the restraining conditions. The resistance of each member to local actions is called the *Member Stiffness*. This is derived from the cross-section stiffness and the geometry of the member. The cross-section stiffness is the resistance of the cross-section to overall strains. This is derived from the cross-section geometry and the stiffness of the materials from which it is made. The material stiffness which is the resistance of the material to strain derived from the stiffness of the material particles. It can be seen from the curves that non-linearity in stiffness comes from the cross-section and material nonlinearity.

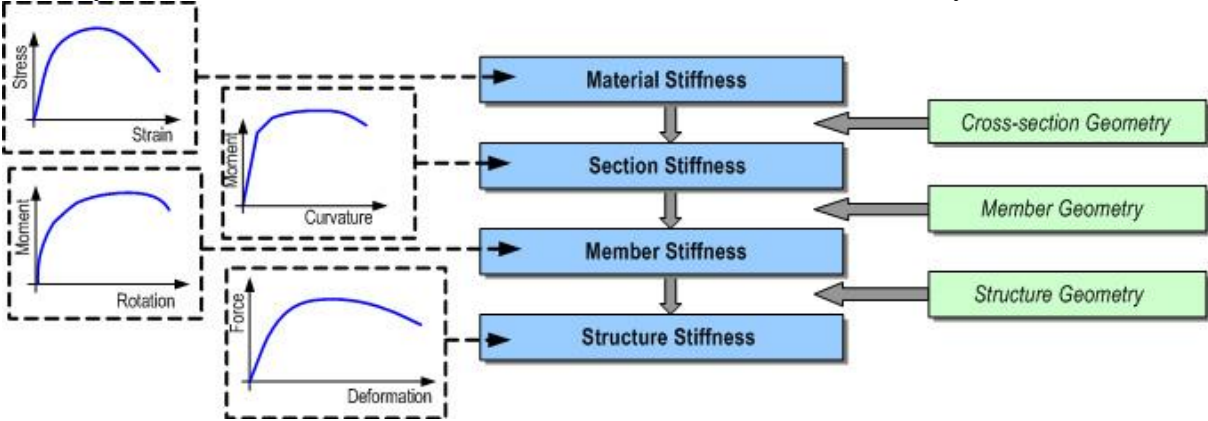


Figure 9: The built up of structural stiffness from the contribution of stiffness at various levels

The product of modulus of elasticity and the moment of inertia are often used together in several expressions of stiffness such as EI or EA. This term, generally called the cross-section stiffness can be determined in two ways. The first method is simply the product of the moment of inertia, *I* and the elastic modulus or modulus of elasticity, *E*. However, for reinforced concrete members, both the moment of inertia and its average modulus of elasticity depend on the amount of reinforcement, the extent of cracking and the level of strain; so the elastic stiffness determined using the product of basic *EI* is only valid in the linear-elastic range. Stiffness can indirectly (and more accurately) be determined, from the slope of the *M-φ* curve. It is obvious that the stiffness of the cross-section depends on the level of axial load and bending moment. Therefore, a particular *M-φ* of curve for a particular level of axial load and direction of bending can be used to determine the stiffness for a specified moment as shown in Figure 10. It is important

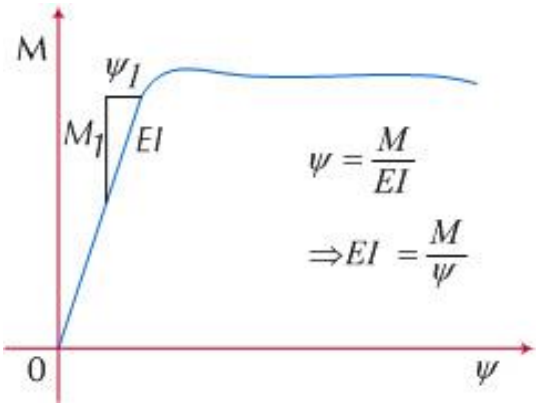


Figure 10: Determination of effective stiffness from the Moment-Curvature

to note that the stiffness determined this way is the "effective stiffness": This includes the effect of cracking, the stiffness contribution due to reinforcement and the change in the modulus of elasticity of concrete and steel due to certain level of strain.

Moreover, if the moment curvature curve has been generated considering the short and long term effects, then the stiffness would automatically be adjusted for all those effects such as confinement, creep, shrinkage, temperature change, etc. The stiffness determined this way can be used directly in structural analysis such as stiffness matrix approach and the linear or non-linear finite element formulations. The stiffness can also be used to determine the deflections using the conventional deflection equations and formulae.

Therefore if the moment curvature curve is used to compute the stiffness, then it automatically includes the material and cross sectional non-linearity. The member and structure level non-linearity in stiffness can be incorporated by using P- $\Delta$  analysis and large displacement analysis while carrying the Pushover analysis as is done in SAP2000 and ETABS [8].

**4.1 Strength Design**

Generally the strength of a reinforced concrete beam-column is defined by a capacity interaction surface. This surface is a plot of applied action (P and M) for all orientations and rotations of the neutral axis plane for a fixed strain of 0.003 or 0.0035 [9]. The actual applied action (P and M) should be within their surface. The surface is further represented by P-M interaction curve for a particular bending direction and  $M_x$ - $M_y$  curve for a given axial load. The P-M interaction is in fact denoted by the yield or capacity point on the Moment-Curvature curve as shown in Figure 11. It is, therefore, possible to generate M- $\Phi$  curve for a given axial load and bending direction and then read off the capacity point to carry out strength design as well. The factors that effect strength include the basic material strength, the cross-section dimensions, the amount of rebars and the overall structural framing conditions. The strength design is a pre-requisite to the performance design and infact required before the Pushover Analysis can be carried out. This is because the cross-section dimension and reinforcement should be known for computation of the hinge properties that are needed for Pushover analysis.

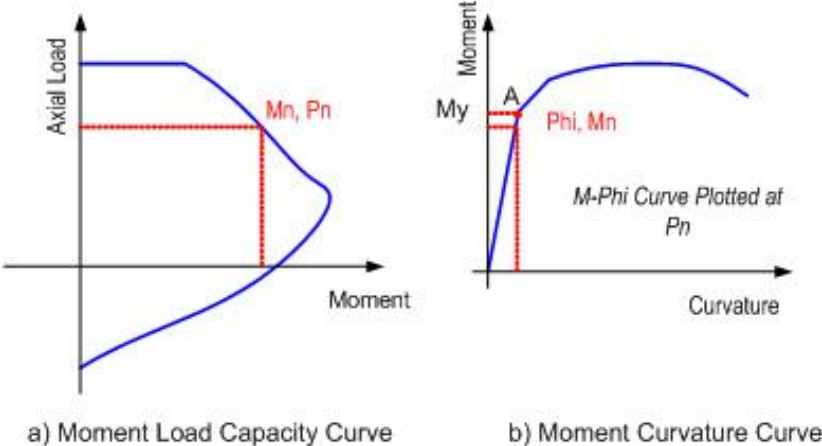


Figure 11: Relationship of capacity point on interaction curve and Moment-Curvature curve

**4.2 Serviceability Design**

The serviceability requirements are generally specified in terms of limit on stresses, limit on deflection and deformations and on crack width or crack spacing. Almost all of these parameters can be obtain directly or indirectly from the Moment-Curvature curve. Figure 12

shows a symbolic relationship of various quantities ranging from strain to deflections. The deflections can be directly computed by using the moment-Curvature curve and the moment diagram due to applied loads.

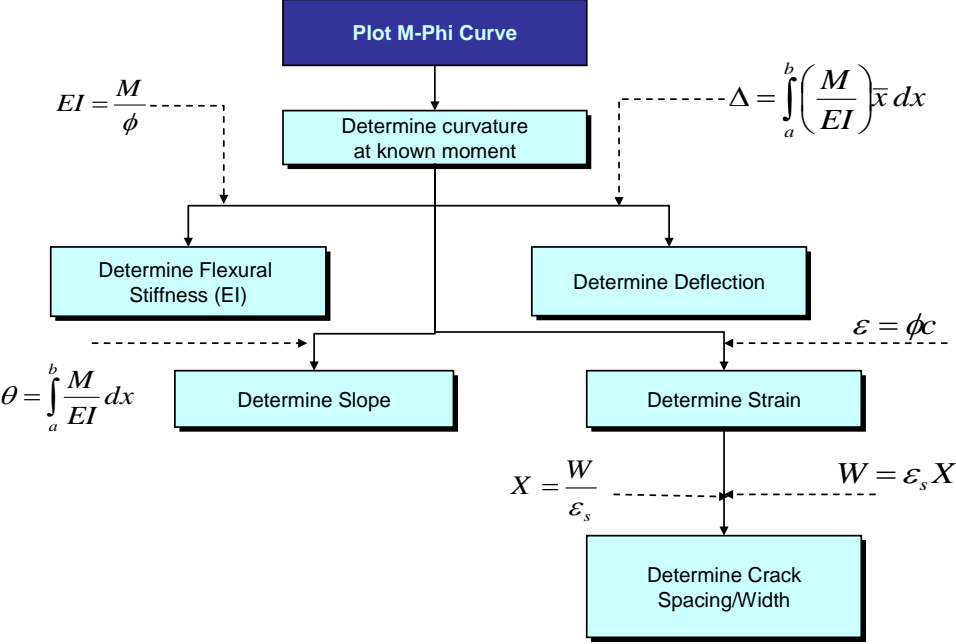


Figure12: The use of moment-curvature curve for computing various quantities

The deflection of a point on a straight member under bending in the direction perpendicular to the original straight axis of the member is equal to the moment of the M/EI diagram between those two points about the point where the deflection occurs.

To compute the deflections by using the Moment-Curvature curves, the following procedure can be used.

1. Design the cross-section for moment and axial load at various locations along the beam-Column.
2. Generate the moment-curvature curves for the designed sections.
3. Plot the moment diagram and the axial load diagram. The axial load is generally constant along the length of the member so it can be used as the controlling variable while generating the Moment-Curvature curves.
4. For various locations along the member length, read the curvature for the applied moment using the appropriate moment-curvature curves. Use interpolation if the cross-section changes along the length.
5. Plot the M/EI diagram along the length of the member.
6. To compute the deflection at a specific location, calculate the area of this M/EI diagram up to that point, starting one end of the member.

It is important to note that the deflection determined this way includes the secondary effect of axial load on change is axial. Flexural stiffness of the Moment-Curvature has been computed for a specific axial load. Although the above procedure can be used manually with certain simplifications, it is best suited for computer applications where the generation of moment-curvature curves, and the integration of M/EI diagram can be automatically

accomplished. Other quantities of interest can also be computed from the Moment-Curvature using the following equations.

Stiffness of the Section at given M and Phi

$$\phi = \frac{M}{EI} \quad 3(a)$$

$$EI = \frac{M}{\phi} \quad 3(b)$$

Slope of the section at given moment

$$\theta = \int_a^b \frac{M}{EI} dx \quad 3(c)$$

Strain at given moment

$$\varepsilon = \phi c \quad 3(d)$$

$c$  = distance from the NA to the point where strain is required

## 5. CONCLUSIONS

The cross-sections and their properties are key components in almost all aspects of theoretical formulation, computer applications, practical analysis and design. The cross section property of general section is made up of several shapes and different materials and requires good theoretical knowledge and aid of computing tools.

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