EXTRACTION OF STRUT-AND-TIE MODELS FROM SHELL ELEMENTS MESH ANALYSIS

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ABSTRACT

Strut-and-tie model (STM) method is a lower bound solution based on the theory of plasticity, which can be used especially for the design of structural concrete members in D-Region. An approach for automatically finding an appropriate strut-and-tie model for planner structural concrete members modeled and analyzed using finite element procedures is introduced in this study. The finite element analysis can be performed in any FEA computer program that has the shell element meshing capability. The shell element force trajectories of the concrete member are obtained and struts and ties are extracted based on the direction and magnitude and graphically displayed. The algorithm includes two main important features: (1) to extract and display an appropriate strut-and-tie model from the output of FEA; and (2) to refine, analyze and design the extracted appropriate strut-and-tie model using local truss analysis. As sample application, concrete deep beam and shear wall configurations are used in demonstrating the capability of the proposed method in finding an appropriate strut-and-tie model and compared with the previous theoretical and experimental studies that deal with STM for the purpose of verification of the results.

Keywords: Strut and Tie Model, Finite element Analysis, Shell element

1. INTRODUCTION

The strut-and-tie model (shortened as STM) method is considered as the basic tool in the analysis, design, and detailing of reinforced concrete members loaded in bending, shear, torsion, [1] and can even consider all of these load effects simultaneously.

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This method evolved as one of the most useful design guiding principles for disturbed or discontinuity regions (D-regions) in concrete structures. The model provides a rational approach of representing the stress flows within structural member with an appropriate truss models, consisting of compressive struts (uniaxial compressive stress fields) and tensile ties (uniaxial tensile stress fields) that represent the actual load transfer mechanism in the concrete member for the applied loads and given support conditions. The original truss analogy concept assumes that after cracking, concrete cannot resist tension stresses anymore, and further postulates that a cracked reinforced concrete member acts as a truss with parallel longitudinal chords and a web composed of diagonal concrete struts and transverse ties [2]. The locations at which struts and ties converge or intersect are called nodes or nodal zones, and these are the locations also at which the forces are redirected within the strut-and-tie model. Nodal zones represent bi-axial (for 2-D problems) or tri-axial (for 3-D problems) stress fields. This study consider only D-regions that can be reasonably assumed as plane (2-D) structures with uniform thickness and the state of stress is principally in-plane (plane stress condition). Additionally, the loading acting in the concrete member is limited to concentrated or point loading, and the only strut-and-tie models that consists of unreinforced struts and non-prestressed reinforcement ties are considered. Furthermore, prismatic shapes for compression struts and tensile ties are employed in the design of the strut-and-tie model.

Once STM’s are developed from any structural concrete members, the model is utilized for an investigation of the equilibrium between the acting loads, the reactions, and the internal forces in the concrete compression struts and in the tension ties. The actual load carried by the model is treated as lower-bound ultimate load for reinforced concrete member based on the lower-bound theorem of plasticity. This approach provides a concise explanation and clear understanding of the behavior of reinforced concrete members, and brings rational and safe design process for any structural concrete members, especially in D-regions.

2. EXTRACTION OF STRUT AND TIE MODEL

2.1 Back Ground

In any structural concrete member such as deep beams, beam-column joints, corbels, pile caps, post-tensioned beams, and shear walls, there is no single and unique STM for most design situations encountered. However, according to Fu [6], there are some techniques and rules which may help the designer to develop an appropriate model. Designers that may use STM method will definitely encounter an iterative process in developing an appropriate STM for a certain structural concrete member, which is a time-consuming process. A so-called evolutionary structural optimization (ESO) method has been developed by Liang et al. [1] based on material removal criteria, however, a time-consuming iterative process in obtaining an appropriate strut-and-tie model remained an unresolved issue. Liang et al. [1] used the ESO method to characterize the continuum topology optimization which is often a truss-like structure. On the other hand, Kwak et al. [3] also adopted the basic idea of ESO method in determining more rational and accurate STM's. In this method, a brick element composed of six truss elements is designed as a basic element unit to prevent the structural instability that may occur during the evolutionary optimization process. Afterwards, a systematic removal of each brick element that has the least virtual strain energy is performed, and then the optimal load transfer mechanism of a reinforced concrete structure is finally characterized on the basis of an optimization criterion of minimizing the total elastic strain energy of the structure. This method thus undergoes also a time consuming iterative process. All of the existing ESO based techniques require specialized tools and computer software either not available or not used widely in practice.

In this study, an iterative process is also introduced where strut-and-tie model can be developed in a quick and less intensive manner by means of applying the basic concept of extraction-and-design approach using the elastic stress distribution. Obtaining such elastic stress distribution within the concrete member is performed through finite element method (FEM), using any existing FEA software tools. FEM is all recognized as the most powerful tool in analyzing complex structure, and most engineers are familiar with the use and application of this method. The basic concept of extraction-and-design approach is a postprocessor and will help to visualize the hidden force transfer mechanics more quickly and clearly, as well as provides the estimation of required reinforcement.

Structural concrete deep beams and shear walls were taken as examples to demonstrate the capability of the method in finding an appropriate STM and corresponding design. These deep beam and shear wall configurations are taken from the previous theoretical and experimental studies in finding STM and are also used for verification of results.

2.2 Proposed Methodology for Extraction of Strut and Tie Model

The first step involved in proposed methodology is the linear elastic analysis of concrete deep beam and shear wall structure using shell element mesh in any software. However, for the purpose of demonstration in this study, SAP2000 V.11 (*version 11*), a state-of-the-art finite element analysis computer program has been used to model and analyze the concrete deep beam and shear wall structures using shell elements (or membrane elements).

In the analysis, proper shell element modeling and meshing of the concrete members has to be implemented to obtain an appropriate results, particularly in obtaining the direct in plane forces *F11* and *F22*, and shearing forces *F12*, as shown in Figure 1, acting at the mid-surface of each shell element. In addition to the forces, information about boundary conditions such as the supports (*hinge, roller, fixed*) and external forces (*concentrated loads*) is also needed.

The axial (*F11* and *F22*) and shearing (*F12*) forces are used to calculate the shell element internal resultant forces (*Fmax* and *Fmin*) and the corresponding angle (θ), in which the path of force trajectory in each shell element can be obtained, as shown in Figure 1. In Figure 1, the connectivity on each quadrilateral shell element from joint 1 (j1) to joint 4 (j4), in a counterclockwise direction, and the corresponding corner joint coordinates are also included in the analysis outputs to be extracted to obtain an appropriate strut-and-tie model. As a summary, following data and information from SAP 2000 (or other FEA software) analysis output database are needed for the extraction of strut-and-tie models; (1) shell element forces at nodes; (2) shell element nodal coordinates; (3) shell element joint connectivity; (4) boundary supports; and (5) boundary forces. These data-base files are exported to the Excel format and read-in by the program developed in this study as the input data after some miner modifications to the format.

The main methodology of STM extraction and design process proposed in this study is implemented through a computer algorithm developed using Microsoft Visual Basic.Net platform as the programming language. The proposed methodology contains two major processes: (1) STM extraction and refinement process; and (2) STM designing process. Following modules are included in the procedure.

i. Import and store FEA output data for Excel format.
ii. Shell elements Normal Vector calculation, to determine the plane of the element.
iii. Principle forces (F_{max} , F_{min}) and Principle angle calculations.
iv. Shell element groupings with nearly obtain equal Normal Vector, to planner models.
v. Shell element groupings with the same principle angle Inclination.
vi. Shell element groupings with nearly equal Fmax (for tie layout)
vii. Shell element groupings with nearly equal Fmin (for strut layout)
viii. Formation of strut and tie model for above groups.

ix. Display of primary strut and tie model

x. Truss member forces evaluation and refining of extracted strut and tie model

xi. Design of truss members for computed force.

Shell element mid point coordinate calculations and coordinate transformation from 3D coordinate system to 2D coordinate system is included in the step of shell element normal vector calculations and shell elements grouping steps follows one after the other in order to find shell element groups which are having approximately same principle forces in magnitude and directions which are assumed to be act in shell element mid points. The calculated mid-point coordinates are used to generate the strut-and-tie lines by performing linear curve fitting using the least square method. This means that the location of the calculated *Fmax* and *Fmin* forces can be found at the midpoint coordinates of each shell element. This generated strut-and-tie lines is displayed in order to view the so-called “primary strut-and-tie model”.

Once the primary truss model is extracted through the algorithm, a truss analysis is carried out to compute truss member forces, which are evaluated and refinement of the truss model is done in order to get optimum strut and tie model. The refinement criteria used for the removal of the truss members which have member forces equal or less than 10% of maximum member force existing in the truss system. Truss member force evaluation is done by implementing the computer algorithm which is considered the static equilibrium among applied loads, support reactions and internal member forces. The designing of strut and tie members are done using truss member forces in extracted optimum truss model by following the recommendations given in the ACI 318-05 Appendix A code provisions.

3. VERIFICATION OF THE PROPOSED METHODOLOGY

As a sample application, concrete deep beam and shear wall configurations are used in demonstrating the capability of the proposed method in finding an appropriate strut-and-tie model and is compared with the previous theoretical and experimental studies that deals with STM.

Three (3) concrete deep beams and two (2) shear walls member with the corresponding variable parameters are utilized. Concrete deep beam members are taken from the previous experimental and computational approaches that deal in STM, on the other hand, an attempt in finding STM in a shear wall is performed in which practical geometry of the wall is used.

The geometry of concrete simply supported deep beam with point loading used by Liang et al[1] in his study of performance based evolutionary optimization is used as first sample application to find the hidden strut and tie model. The present study, using linear elastic FEA under similar condition produces similar topology of strut and tie model as obtained in the reference (Fig 6).

The second sample application is carried out for simply supported deep beam configuration with two point loading used by Tjhin and Kuchma [20] in their study and comparison of the results is showing in Fig8 and 9.

The third deep beam configuration considered in the study is taken from the Rogowsky's study. In which simply supported two span deep beam configurations with two point loads acting on it are considered as shown in Fig 10. All of these comparisons indicate that the methodology used in the present study can reasonably determine the strut and tie configuration based on the results of linear static finite element analysis.

The second approach is applied to the extraction of strut and tie configuration in a rather tall wall in which the "D" region is near the top where a concentrated load is applied. The approach used in the study correctly identified a truss mechanism at the top and more "flexure" C-T combination in the flexural on "B" region.

4. SUMMARY AND CONCLUSION

This study introduced an approach in finding the strut-and-tie model (STM) for concrete deep beam and shear wall member. In this approach, STM is extracted from the linear-elastic analysis using shell element mesh. Such analysis can be performed in any FEA software, though SAP2000 V.11, a state-of-the-art FEA program is used as a sample. The analysis is carried out in order to obtain the following required results: (1) shell element forces at nodes (direct and shearing forces acting at the mid-surface of the shell element); (2) shell element joint connectivity; and (3) shell element joint coordinates. Additionally, (4) boundary supports and (5) forces (concentrated loads) are also needed for the extraction of STM.

The obtained results and other information from FEA are then imported into a computer program that implements an algorithm to extract, display, refine, analyze, and design an STM. The first extracted model is the so-called "primary extracted strut-and-tie model", and needs to be refined to obtain an appropriate STM. The refinement process is through a local analysis in obtaining member forces on truss elements. An element with 10% of the maximum member force or less is removed, and then re-analyzed until no elements with less force are present in the model. Thus, an appropriate STM is obtained and displayed. The extracted appropriate STM is then designed to check the concrete capacity of compressive struts and the amount of reinforcement needed in the tensile ties. Design code provision of ACI 318-05 Appendix A are used in the design of the extracted appropriate STM.

There are three (3) concrete deep beams and two (2) shear walls used to demonstrate the capability of the proposed method in finding an appropriate STM. Deep beams were taken from different reference that deal in finding STM, while shear wall were taken in order to test the capability of the proposed methodology. Based on the results, the following conclusions were drawn:

1. Finite element analysis using shell element mesh in SAP2000 computer program can provide initial layout and model of both strut and tie;

2. The proposed method can find an appropriate STM that characterizes the internal stress flows in concrete deep beam and shear wall member;
3. The algorithm developed in Microsoft Visual Basic.Net platform can extract and design an appropriate STM from the results of finite element analysis using shell element mesh;
4. The proposed algorithm can handle both planar and non-planar walls and beams, although the results have not been verified at this stage for non planner shear walls.
5. The proposed algorithm can serve as a post processor for determining and design using strut and tie models, and can also identify the truss configuration that may be introduced back in the full FEA models for system level analysis containing several deep beams and walls.
6. The approach is based on linear static elastic analysis and does not explicitly consider the plasticity or nonlinearity effects.

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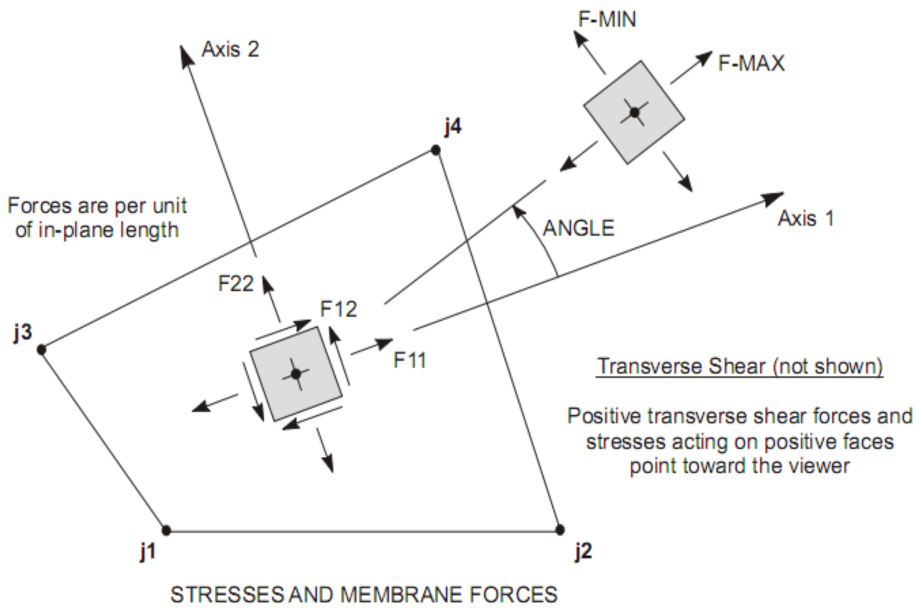
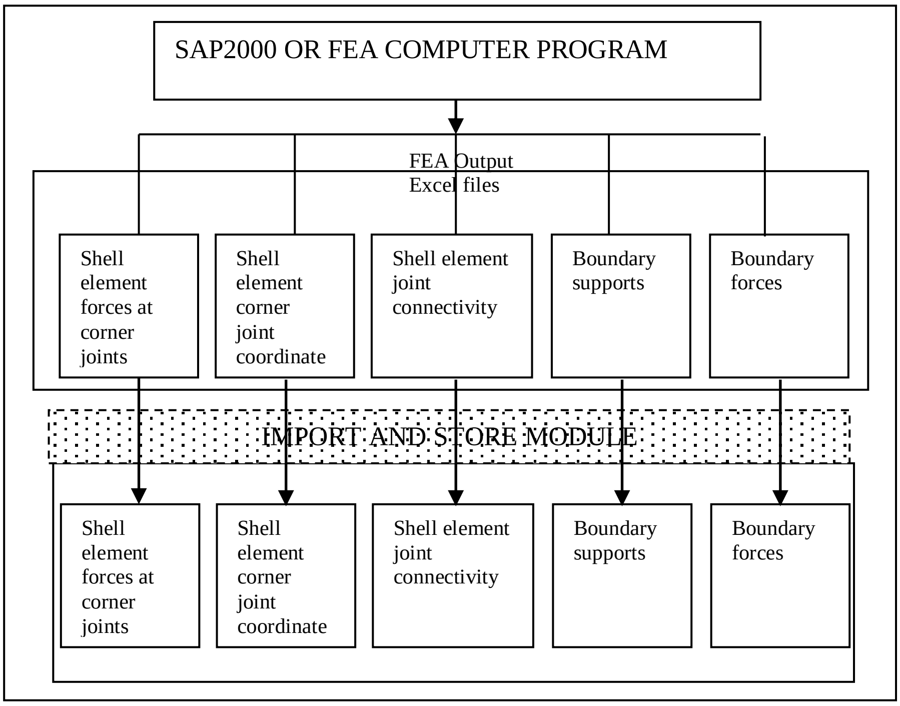


Fig 1. Shell element internal forces (SAP2000 Reference Manual)



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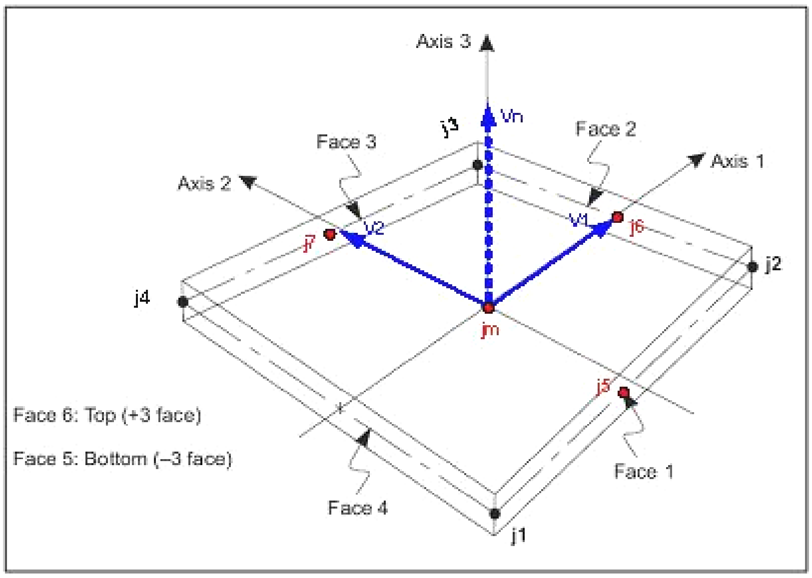

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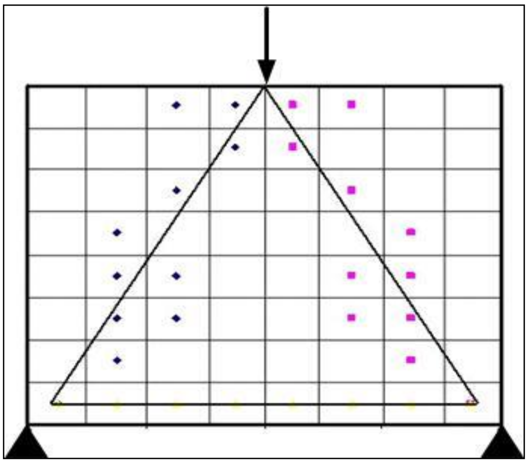

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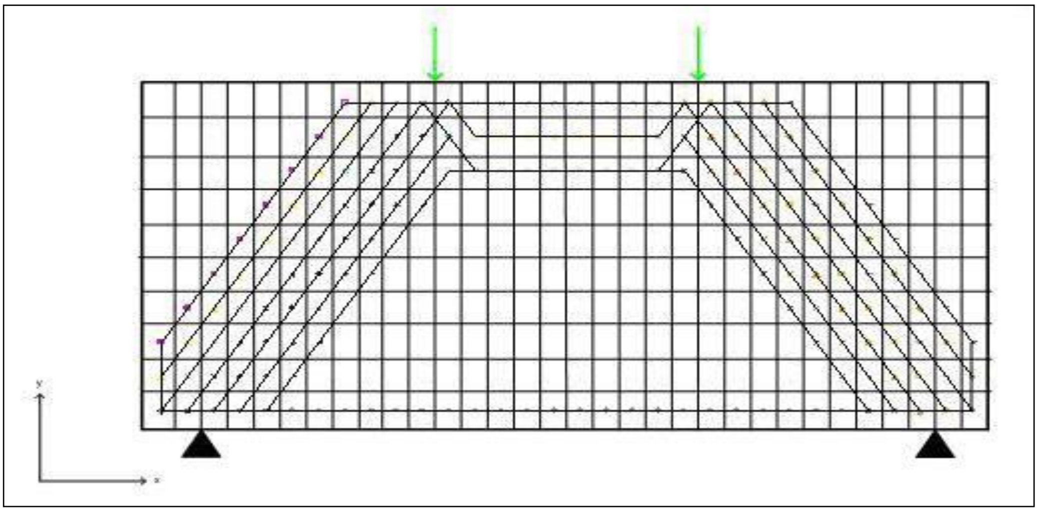

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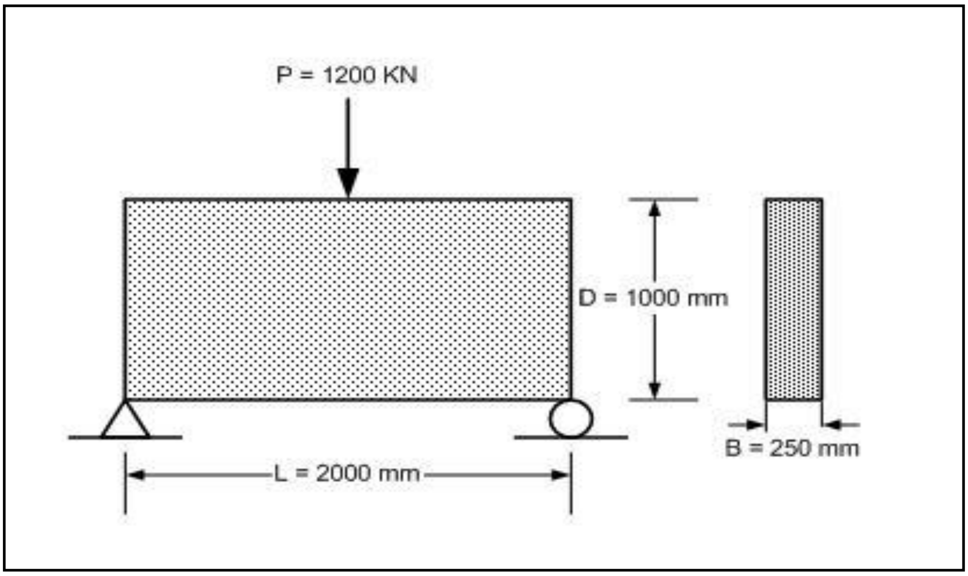
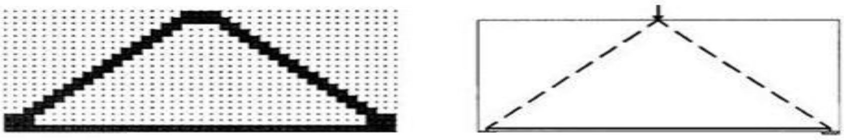


Fig6. Single-span concrete deep beam configuration under one-point loading taken from Liang et al. (2000)



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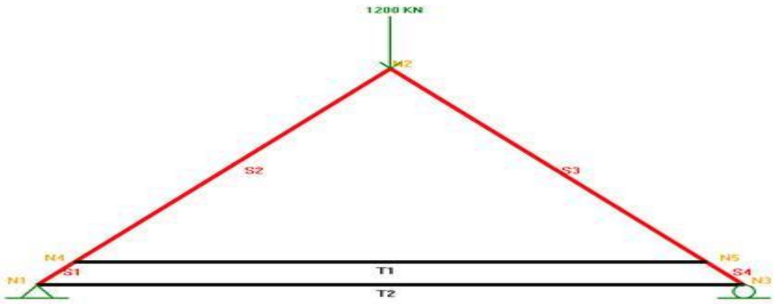

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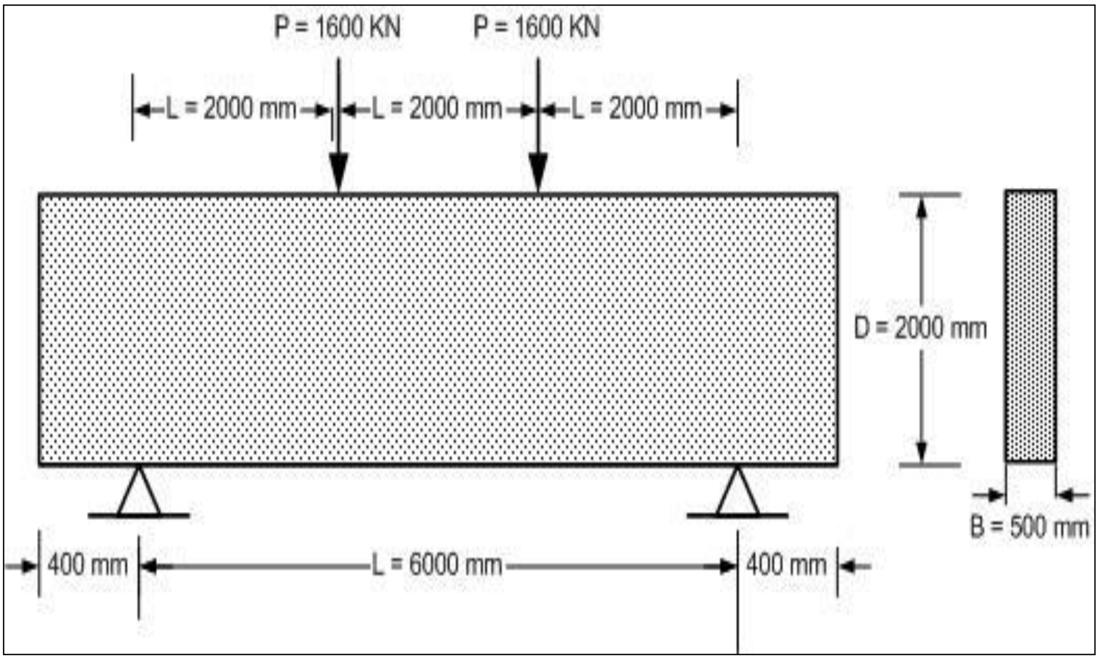

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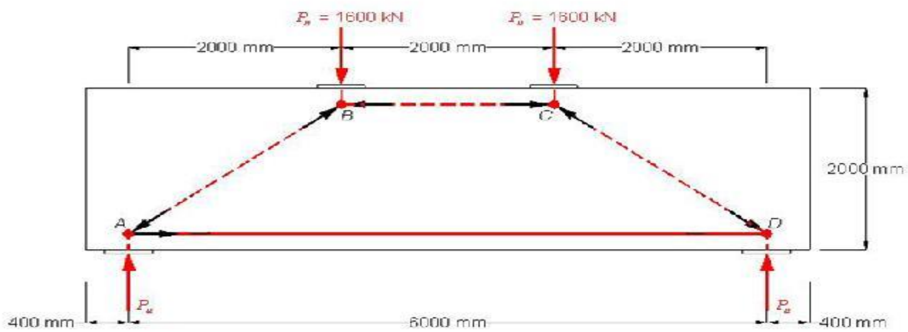

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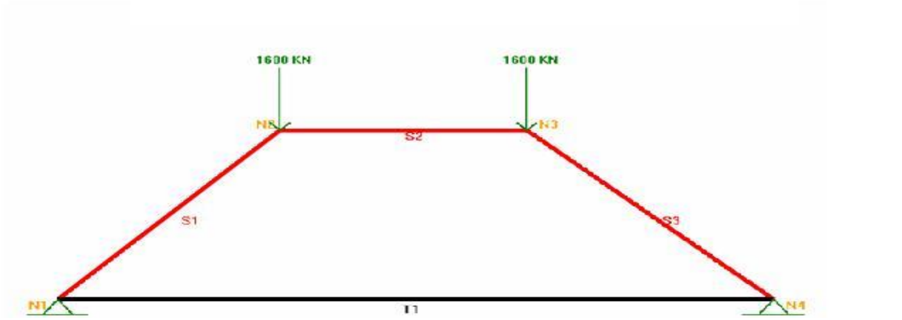

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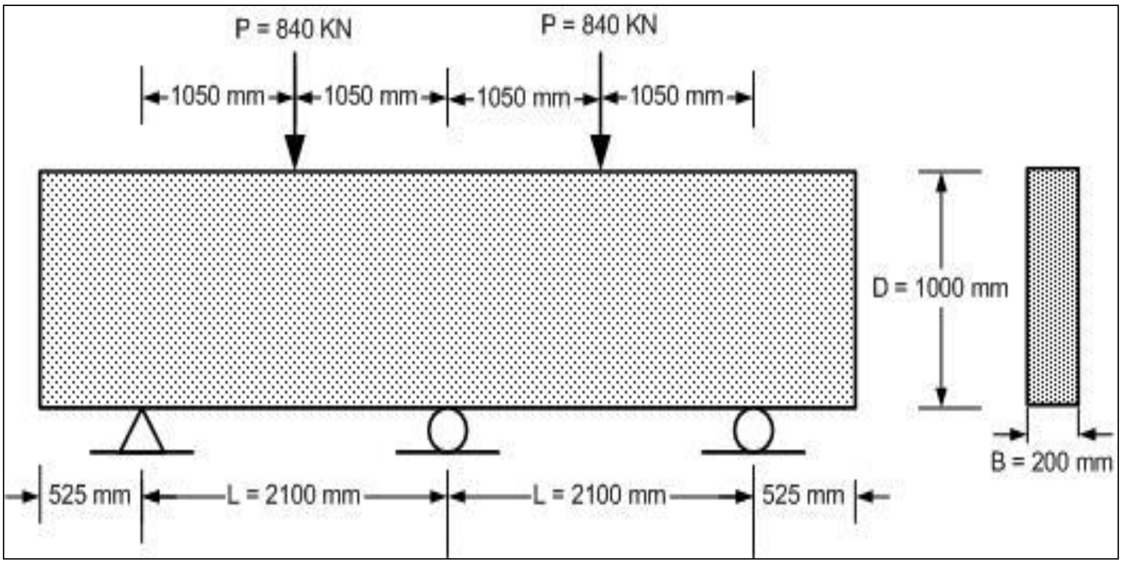

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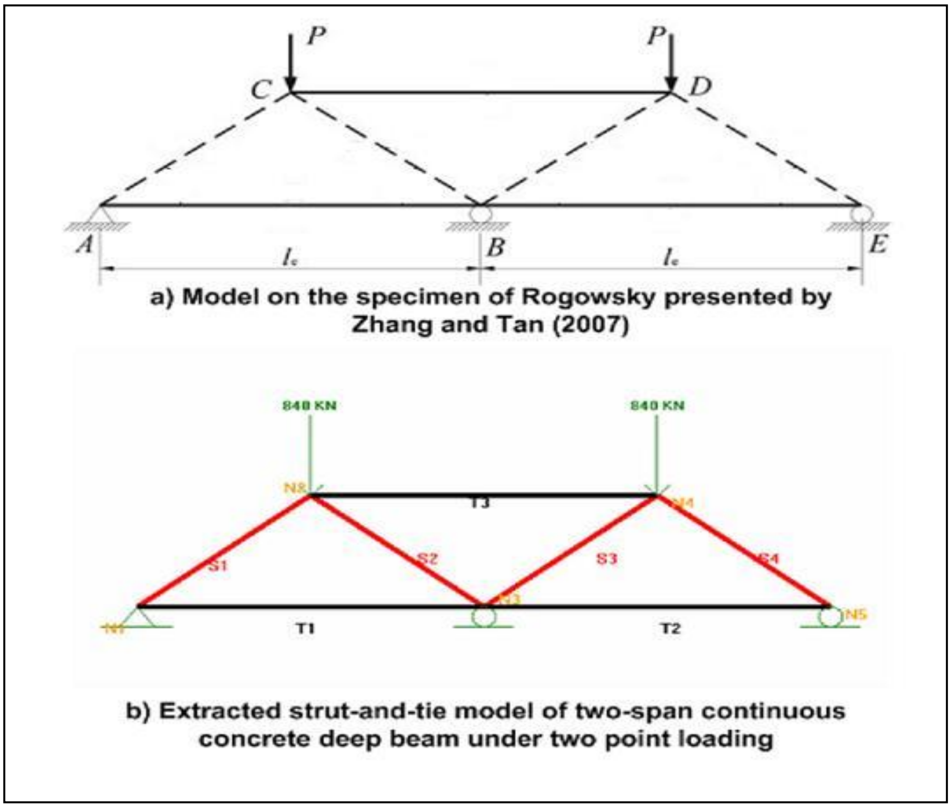

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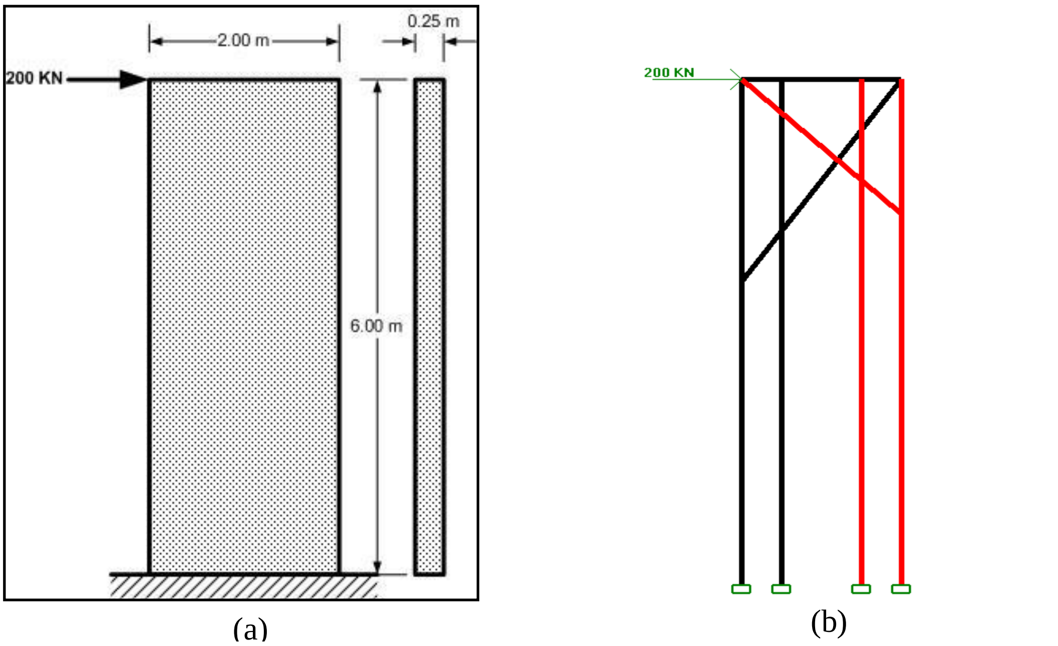

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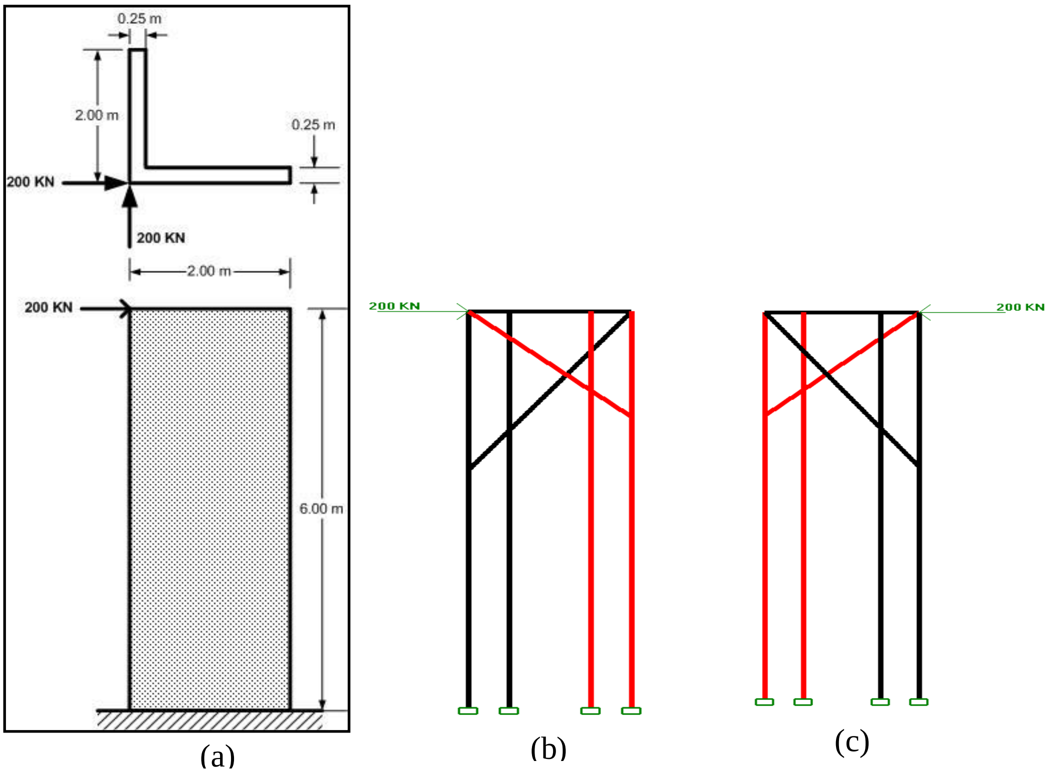

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